Unit 2 Set Theory

2.0 Unit Objectives:

By the end of this Unit, learners should be able to:

- Understand concept of a set.
- Describe different types of sets.
- Perform various operations on sets.
- Explain different properties of sets.
- Represent sets using Venn diagrams.

2.1 Unit Introduction:

Set Theory is the mathematical science in which properties of sets are studied. Sets are basic abstract objects that are used in the study of logic, discrete mathematics and computer science etc. Here we will discuss the basics of set, types of sets and different operations on sets.

2.2 Set notations:

Definition 2.2.1 **Set**: *A set is a well defined collection of objects*.

The objects of a set are called elements or members.

The elements of a set can be anything. A set can be a collection of numbers, collection of names of persons, collection of letters of the alphabet or even collection of other sets. Sets are conventionally denoted with capital letters *A*, *B*, *C*, etc. And the elements are generally denoted with lower case letters x, y, z etc. If x is a member of a set A, then symbolically it is denoted as $x \in A$, and if x is not a member of a set A then symbolically it is denoted as $x \notin A$.

Various methods are used to indicate members of a set. The two methods which are more commonly used to represent sets are the Set listing method and the Set builder method.

1: <u>Listing Method / Roaster Method:</u>

In set listing method the elements of a set are explicitly listed within braces or curly brackets. If set contains only a small number of elements or if there is some particular pattern in the list of elements then this method is useful.

The order in which the elements of a set are listed is not important. And also the repetitions in the list do not change the contents of a set.

For example, {2,4,6, 8} = {6,2,8, 4 } = {8,6, 4, 2} = {2,2,4,6,6,8}. For the sets with too many elements, more often an abbreviated list is used. This list is ended with three dots "**…**", which indicates that the list continues in the obvious way.

Examples:

- A = {1, 3, 5,7, 9}. Here 1, 3, 5,7 and 9 are the members of the set A.

  So we can write $1 \in A$, $3 \in A$, $5 \in A$, $7 \in A$ and $9 \in A$; but $2 \notin A$, $4 \notin A$ or $8 \notin A$ etc.
- B = {BASIC, C, C++, PASCAL}

  In this case BASIC, C, C++ and PASCAL are the members of set B. So we can write BASIC $\in$A, C$\in$A, C++ $\in$A , and PASCAL $\in$A whereas COBOL$\notin$ A or FORTAN $\notin$ A etc.
- The set of the first one hundred natural numbers can be described symbolically using listing method as , A = {1, 2, 3, ..., 100}.
- Similarly the set of all odd natural numbers can be written using listing method as {1, 3, 5, 7, ... }.

Note that if we cannot observe any particular pattern in the list of members and we cannot guess at any stage what the next number in the list might be, then for such sets listing method is not useful.

2. Set builder Method: More complicated sets are described by a different way. If a set contains a large number of elements and the members of the set follow some obvious pattern, then the set builder method is used to represent such set. In this method a set is described by indicating the properties that its members must satisfy.

The generalized form set-builder notation is $\{x : p(x)\}$. It denotes the set of every object *x* which satisfy the property p(x). In this description, the colon "**:**" means "such that" (Sometimes the notation "**|**" is used instead of the colon.).

Examples:

- A = { x : x is a natural number less than 101} . This we read as the set A is a set of all x, such that x is a natural number less than 101.

  This set A contains hundred numbers which are all natural numbers from 1 to 100. This set can also be represented using listing method as A = {1, 2, 3, **...**, 100}.
- B = $\{x\textbf{:}\ x$ is a natural number and $2 \le x^2 \le 30\}$. This we read as the set B is a set of all x, such that x is a natural number whose square is between 2 to 30. This set A can be written using set listing method as B = { 2, 3, 4, 5 }.

❖ ***<u>Self Test I</u>*: Select the correct alternative from the given alternatives.**

1. Which of the following represents the statement "The number 5 is not a member of the set A"?

(a) $5 \in A$ **(b)** $5 \notin A$ **(c)** $A \notin 5$ **(d)** $A \in 5$.

2. Which of the following represents the statement "The number 10 is a member of the set B"?

(a) $10 \in B$ **(b)** $10 \notin B$ **(c)** $B \notin 10$ **(d)** $B \in 10$.

3. Which of the following represents the set A = {11, 13, 15, 17, 19 }?

(a) A = { x : x is a natural number greater than 11}

(b) A = { x : x is an odd natural number greater than 11}

(c) A = { x : x is an odd natural number between 10 to 20 }

(d) A = { x : x is a natural number less than 20} .

4. Which of the following represents the set A = {1, 4, 9, 16,25 }?

(a) A = { x : x is a square of natural number less than 30}

(b) A = { x : x is an odd natural number less than 30 }

(c) A = { x : x is an odd natural number between 1 to 30 }

(d) A = { x : x is a natural number less than 30} .

5. Which of the following represents the set A = {a, e, i, o, u }?

(a) A = { x : x is alphabet } **(b)** A = { x : x is English alphabet }

(c) A = { x : x is an English alphabet and a vowel}

(d) A = { x : x is an English alphabet and a consonant } .

6. Which of the following represents the set A = {4, 5, 6, 7, 8, 9 }?

(a) A = { x : x is a number less than 10} **(b)** A = { x : x is an integer greater than 3}

(c) A = { x : x is an odd integer between 3 to 10 }

(d) A = { x : x is an integer and $3 < x < 10$} .

7. Which of the following represents the set ,B = { x : x is an integer and $3x = 6$ } ?

(a) B = { } **(b)** B = { 2 } **(c)** B = { 3 } **(d)** B = { 6 }.

8. Which of the following represents the set, B = { x : x is an integer, $x^2 + 1 = 10$} ?

(a) B = { – 3 ,…, 3} **(b)** B = { – 3 , 3} **(c)** B = { 3} **(d)** B = { }.

9. Which of the following represents the following set ,

B = { x : x is an even natural number greater than 25 and less than 35 } ?

(a) B = { 26, 27, 28,…, 34} **(b)** B = { 26, 28, 29, 30…, 34}

(c) B = { 24,…,34} **(d)** B = { 26, 28, 30,32, 34}.

10. Which of the following represents the following set,

B = { x : x is a vowel and x is not a or i } ?

(a) B = { a, e, i, o, u} **(b)** B = { a, i } **(c)** B = { e, i, o, u} **(d)** B = { e, o, u} .

2.3 Types of sets :

Definition 2.3.1: Empty Set or Null set:

The empty set is a set which contains no elements.

Usually empty set is represented using the symbol ϕ. Some times it is also represented as { }. But note that the set $\{\phi\}$ is not an empty set.

Examples:

- A is the set of natural numbers whose squares are negative. This set A is an empty set because there is no natural number whose square is negative.
- B = { x : x is a prime integer whose square is one}. This set B = ϕ, because there is no prime integer whose square is one.

Definition 2.3.2: Singleton Set: *A singleton set is a set which contains only one element.*

Examples:

- If A is the set of all natural numbers whose square is 100, then set A = { 10}. A is a singleton set because there is only one natural number whose square is 100.
- B = { x : x is an even prime integer} . This set in listing form can be written as B = { 2}. B is a singleton set because there is only one prime number which is even.

Definition 2.3.3: Subset and Superset:

If every member of the set A is also a member of the set B, then set A is said to be a subset of set B. It is written as A $\subset$ B.

It is also pronounced as "*A is contained in B*".

Equivalently, in this case we can write B $\supset$ A, and read it as "*B is a superset of A*", or "*B includes A*", or "*B contains A*".

Empty set is a subset of every set.

If *A* is a subset of but not equal to *B*, then *A* is called a proper subset of *B*, it is written as A $\subset$ B. In this case *B is a proper superset of A and* it is written as B $\supset$ A.

Examples:

- The set of all women is a proper subset of the set of all people.
- $\{2, 5, 7\} \subset \{1, 2, 5, 6, 7\}$
- $\{a, b, c, d, e\} \subseteq \{a, b, c, d, e\}$. In fact every set is a subset as well as superset of itself.

- {black, yellow, blue, red} $\supset$ { red, yellow}

For any set A empty set is a subset of A and the set A itself is also a subset of A. These two subsets are called improper subsets of set A and all remaining subsets are called proper subsets of set A.

Example:

- *If A* = {a, b, c} then, the improper subsets of *A* are Ø (the empty set) and A= {a, b, c} whereas the proper subsets of *A* are {a}, { b}, {c}, {a, b}, {a, c} and {b, c}.

Definition 2.3.4: Finite set and infinite set : *A set is said to be a finite set if it contains a finite number of elements, all other sets are called infinite sets.*

Examples:

- *A* = {*x*: *x* is a natural number and $2 \le x^2 \le 30$ } .It is a finite set because it contains finite number of members which are 2 ,3,4 and 5.
- B = {a, b, c, d, e, f }.It is also a finite set because it contains 6 members only.
- C = { x : x is a prime integer}. This set is an infinite set because the number of prime integers is not finite. Using set listing form this set can be written as C = {2, 3, 5, 7, 11, **…**}

Some standard infinite sets of numbers, which are commonly used, are listed below:

- Set of all natural numbers, $\mathbb{N}$ = { 1, 2, 3, **…**}
- Set of all integers, $\mathbb{Z}$ = { **…** –3, –2 , – 1, 0, 1, 2, 3, **…**}
- Set of all rational numbers, $\mathbb{Q} = \{ p / q : p , q \in \mathbb{Z}, q \neq 0 \}$
- Set of all real numbers, $\mathbb{R} = \{ x : -\infty \le x \le \infty \}$

Definition 2.3.5: Universal Set**.** *A set which contains all the elements in the universe of discourse is called a universal set.*

It is generally denoted by *U*.

Normally anything under consideration is part of the Universal Set. So any thing other than the universal set is an empty set.

Examples:

- If the sets involved in discussion are of numbers then the universal set can be $\mathbb{R}$, the set of all real numbers.
- If we are discussing about people in different states of India, then the universal set is the set of all people in India.

Definition 2.3.6: <u>Power set</u>**:**

*The set of all subsets of a set **A** is called the power set of **A** and is denoted by* $\mathcal{P}$(A) **.**

If a set A contains n number of elements then its power set $\mathcal{P}$(A) contains 2^n number of elements.

Examples:

- If A = { 2,3} then all possible subsets of A are ϕ (the empty set), {2}, {3} and {2,3}. Therefore the power set of *A* is $\mathcal{P}$(A)= {ϕ, {2}, {3}, {2, 3} }.

We observe that set A contains 2 elements and $\mathcal{P}$(A) contains 4 = 2^2 elements.

- If *B* = {a, b, c} then all subsets of *B* are as follows : ϕ, {a}, {b}, {c},{a, b}, {a, c}, {b, c} and {a, b, c} .

Therefore the power set of *B* is $\mathcal{P}$(B)= {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c} }.

As the set B contains 3 elements, its power set contains 8 = 2^3 elements.

❖ ***<u>Self Test II</u>*:**

Select the correct alternative from the given alternatives.

1. Which of the following is a null (or empty) set?

(a) { x : x is a natural number and $x^2+1 = 10$}

(b) { x : x is a natural number and $x^2 = 121$}

(c) { x : x is a natural number and $x^2 = -10$ }

(d) { x : x is a natural number and $x^2 \leq 100$ } .

2. Which of the following is a singleton set?

(a){ x : x is an integer and $x^2 = 16$} **(b)** { x : x is an integer and $x^2 - 1 = 120$}

(c) { x : x is a an integer and $x^2 = x$ } **(d)** { x : x is a an integer and 4x = 8} .

3. If A = {1, 2, 3, 4, 5} and B = {2, 4, 5 }, then which of the following holds?

(a) $A \subset B$ **(b)** $B \subset A$ **(c)** A = B **(d)** $A = B^c$.

4. Which of the following sets is a finite set?

(a) { x : x is an integer and $x^2 > 0$} **(b)** A = { x :x is a prime number greater than 10}

(c) { x : x is a an integer and $x^2 = x$ } **(d)** A = { x : x is an integer less than 20} .

5. Which of the following is true for the standard sets?

(a) $\mathbb{R} \subset \mathbb{Z}$ **(b)** $\mathbb{Z} \subset \mathbb{Q}$ **(c)** $\mathbb{Z} \subset \mathbb{N}$ **(d)** $\mathbb{Q} \subset \mathbb{N}$.

6. Which of the following can be an Universal set, for the sets A = {1, 4, 9, 16,25 }, B = { 3, 7, 20} and C= { 15, 20, 25} ?

(a) U = { x : x is a square of natural number less than 30}
(b) U = { x : x is an odd natural number less than 30 }
(c) U = { x : x is an odd natural number between 1 to 30 }
(d) U = { x : x is a natural number less than 30} .

7. If we are dealing with the set of all computer programmers in the world, then which of the following can be an Universal set ?
(a) set of all men in the world
(b) set of all women in the world
(c) set of all people in the world
(d) set of all Indians in the world.

8. If A = {4, 5, 6, 7,9 }, then power set of A contains how many elements?
(a) 4 **(b)** 25 **(c)** 5 **(d)** 32

9. If A = {1, 3, 9 }, then which of the following is power set of A ?
(a) { 1, 3}, {3, 9}, , {3, 1}, {9, 3}, {1, 9} }
(b) { { } , { 1}, { 3 }, { 9 }, {1, 3}, {1, 9}, {3, 9}, {1, 3, 9} }
(c) { ϕ, {1, 4}, {1, 9},{3, 7} , A } **(d)** B = { {1, 3}, {3, 9} }.

10. Which of the following represents the power set of $\mathbb{N}$?
(a) { {1, 1}, {1, 2}, {1, 3} ,**…**} **(b)** { A : A is a subset of $\mathbb{N}$ }
(c) { x : $-\infty \le x \le \infty$ } **(d)** { {x } : x is a member of $\mathbb{N}$ }.

2.4 Set Operations

In arithmetic we study different operations of two number such as addition, multiplication, division etc. Similarly we define different operations on sets which are union, intersection, subtraction Cartesian products etc.

Definition 2.4.1: <u>Equality of sets:</u> *Two sets A and B are said to be equal, if they contain the same elements.* It is written as A = B.

Any two sets ***A*** and ***B***, are equal if and only if all members x of set A are such that x is also a member of set B. So if it is true that, A $\subseteq$ B and B $\subseteq$ A, then A= B.

Examples :

- {1, 2, 3 } = {3, 2, 1} ,
- If *A* = {*x***:** *x* is a natural number and $2 \le x^2 \le 30$ }

and B = { 2,3, 4, 5 } then we observe that every member of set A is also a member of set B and every member of set B is also a member of set A. Therefore A = B.

Definition 2.4.2: <u>Union of sets:</u> *If A and B are any two sets, then the union of sets A and B, is the set of all the elements which are either from set A or from set B or from both sets.* It is denoted by $A \cup B$.

Symbolically this set is defined as $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

Examples :

- If $A = \{1, 2, 3\}$ and $B = \{1, 2, 4, 5\}$,then $A \cup B = \{1, 2, 3, 4,5\}$.
- If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$,then $A \cup B = \{1, 2, 3, 4,5 \}$.

Definition 2.4.3: <u>Intersection of sets:</u> *If A and B are any two sets, then the intersection of sets A and B, is the set of all elements which are in both A and B (i.e. those elements which are common to both sets).* It is denoted by $A \cap B$.

Symbolically this set is defined as $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$.

Examples:

- If $A = \{1, 2, 3, 7 \}$ and $B = \{1, 2, 4, 5 \}$,then $A \cap B = \{1, 2 \}$.
- If $A = \{1, 2, 3\}$ and $B = \{4, 5 \}$, then $A \cap B = \phi$.
- If $A = \{1, 2, 3 \}$ and $B = \{1, 2, 3 \}$,then $A \cap B = \{1, 2, 3 \}$.

If two sets A and B have no common elements then $A \cap B = \phi$, such sets are called mutually disjoint sets.

Definition 2.4.4: <u>Difference of sets:</u> *If A and B are any two sets, then the difference of set A from set B , is a set of all those elements of A which are not elements of B.* It is denoted by $A - B$ (or A / B).

Symbolically this set is defined as $A - B = \{ x \mid x \in A \text{ and } x \notin B\}$.

And similarly $B - A = \{ x \mid x \in B \text{ and } x \notin A\}$ .Obviously $A - B \neq B - A$.

Examples:

- If $A = \{1, 2, 3,7,9 \}$ and $B = \{1, 2, 3,4, 5\}$,then $A - B = \{7,9 \}$
  and $B - A = \{4, 5 \}$.
- If $A = \{1, 2, 3,\ldots 9 \}$ and $B = \{1, 3, 5, 7, 9\}$,then $A - B = \{2, 4, 6,8 \}$
  and $B - A = \phi$.

Definition 2.4.5: <u>Complement of a set:</u> *If A is any set for which U is the universal set, then the Complement of the set A is the set which contains those elements of U which are not elements of A.* It is denoted by A^C *or* A'.

Symbolically this set is defined as $A^C = A' = \{ x \mid x \in U \text{ and } x \notin A\}$. Obviously $A^C = U - A$.

Examples :

- If $U = \{1, 2, 3, \ldots, 10 \}$ and $A = \{1, 2, 3,4, 5 \}$,then $A^C = \{6, 7, 8, 9, 10 \}$.
- U is the set of all integers and B is the set of all even integers then B^C is the set of all odd integers.

For every set A, there are no common elements in sets A and A^C ,i.e. $A \cap A^C = \phi$. So every set and its compliment are disjoint sets.

Definition 2.4.6: Cartesian product: *If A and B are any two sets, then the set of all ordered pairs (a, b), where a is an element of A and b is an element of B, is called the Cartesian product of A and B. It is denoted by A X B.*

Cartesian product of two set A X B is formally defined as

A X B= { (a, b) | (a $\in$A and b$\in$B }. Similarly the Cartesian product of set B with set A is $B \times A$ = { (b, a) | b $\in$ B and a $\in$ A} .Obviously $A \times B \neq B \times A$

Examples :

- If A = {1, 2} and B = {a, b, c }, then
  A X B = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c) } and
  B X A = { (a, 1), (b, 1), (c, 1), (a, 2), (b, 2), (c, 2) }
- If A ={x, y, z} , then
  A X A = { (x, x), (x, y), (x, z), (y, x), (y, y), (y, z), (z, x), (z, y), (z, z)}

❖ ***Self Test III*:**

Select the correct alternative from the given alternatives.

1. If A = {2, 3, 5,7,11, 13 }, then which of the following B is such that, A = B ?

(a) B = { x : x is a natural number between 1 and 13}

(b) B = { x : x is a natural number less than 13}

(c) B = { x : x is a prime number less than 15 }

(d) B = { x : x is a natural number greater than 1 }

2. If A = {1, 2, 3, 4, 5 }, B = { 1,3,5,7,9}, then which of the following is AUB ?

(a) { 1, 2, 3, 4, 5, 7, 9 } **(b)** {1, 2, 3, 4, 5 }

(c) { 1,3,5,7,9} **(d)** { 1, 3, 5 }.

3. If A = { x : x is an even natural number between 1 to 11 } and B = { x : x is a prime number less than 15 }, then which of the following is AUB ?

(a) { 1, 2, 3, 4, 5,7, 9 } **(b)** { 2, 4, 6,8, 10}

(c) { 2, 3, 4, 5, 6, 7,8, 10, 11, 13, } **(d)** {2, 3, 5,7,11, 13 }.

4. If A = {1, 2, 3, 4, 5 }, B = { 1,3,5,7,9}, then which of the following is $A \cap B$?

(a) { 1, 2, 3, 4, 5,7, 9 } **(b)** {1, 2, 3, 4, 5 }

(c) { 1,3,5,7,9} **(d)** { 1, 3, 5 }.

5. If A = { x : x is an odd natural number between 1 to 11 both inclusive} and B = { x : x is a prime number less than 15 }, then which of the following is $A \cap B$?

(a) { 3, 5,7, 11 } **(b)** { 1, 2, 3, **…**, 11 }

(c) { 1, 3, 5,7,9, 11} **(d)** {2, 3, 5,7,11, 13 }.

6. If A = {1, 2, 3, **…**, 10}, B = { 1,4,9, 16, 25}, then which of the following is A – B ?

(a) { 1, 2, 3, 5,7, 16 } **(b)** {1, 2, 3, 5,6,7,8,10 }

(c) { 1, 3,5,7,9} **(d)** { 16, 25 }.

7. If A = { x : x is a natural number between 5 to 15 both inclusive} and B = { x : x is a prime number less than 18 }, then which of the following is B – A?

(a) { 6, 8, 9, 10, 12, 14, 15, 16 } **(b)** { 5, 6, 7, **…**, 15 }

(c) { 1, 3, 5,7,9, 11} **(d)** {2, 3, 17 }.

8. If universal set is the set of all people in the world and A is the set of all Indians in the world, then which of the following is A^c?

(a) Set of all Americans in the world

(b) Set of all persons in the world who are not Indians

(c) Set of all people in the world

(d) Set of all Non Resident Indians in the world.

9. If A = {1, 4, 9 }, B = { 3, 7}, then which of the following is AXB ?

(a) { (3, 1), (3, 4), (3, 9), (7, 1), (7, 4), (7, 9) }

(b) { (1, 3), (1, 7), (4, 3), (4, 7) (9, 3), (9, 7) }

(c) { (1, 4), (1, 9),(3, 7) } **(d)** B = { (1, 3), (4, 7) }.

10. Which of the following represents the Cartesian product $\mathbb{R}$ x $\mathbb{R}$?

(a) { (1, 1), (1, 2), (1, 3) ,**…**} **(b)** { (x, x) : $x \in \mathbb{R}$ }

(c) { x : $-\infty \leq x \leq \infty$ } **(d)** { (x, y) : $x \in \mathbb{R}$ and $y \in \mathbb{R}$ }.

2.5 : Properties of set operations:

The operations on sets satisfy many algebraic properties. All of these properties can be proved using the definitions of the operations. These can also be proved using the diagrammatic representations of sets which are Venn diagrams. We will study these Venn diagrams later.

All above defined operations on sets satisfy the following properties:

1. Commutative properties:

For all sets A and B, we have

(i) Union of sets is a commutative operation, i.e. $A \cup B = B \cup A$.

(ii) Intersection of sets is a commutative operation, i.e. $A \cap B = B \cap A$.

2. Associative properties:

For all sets A, B and C, we have

(i) Union of sets is an associative operation, i.e. $(A \cup B) \cup C = A \cup (B \cup C)$.

(ii) Intersection of sets is an associative operation, i.e. $(A \cap B) \cap C = A \cap (B \cap C)$.

3. Distributive properties:

For all sets A, B and C, we have

(i) Union of sets is a distributive operation over intersection of sets

i.e. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(ii) Intersection of sets is a distributive operation over union of sets

i.e. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4. Idempotent laws:

For every set A, we have

(i) $A \cup A = A$.

(ii) $A \cap A = A$.

5. DeMorgan's laws:

For all sets A and B, we have

(i) $(A \cup B)^c = A^c \cap B^c$.

(ii) $(A \cap B)^c = A^c \cup B^c$.

6. Properties of complements:

For every set A , universal set U and null set ϕ , we have

(i) $((A)^c)^c = A$.

(ii) $A \cup A^c = U$.

(iii) $A \cap A^c = \phi$.

(iv) $\phi^c = U$.

(v) $U^c = \phi$.

7. Properties of the universal set:

For every set A and universal set U , we have

(i) $A \cup U = U$.

(ii) $A \cap U = A$.

8. Properties of the null set:

For every set A and null set ϕ, we have

(i) $A \cup \phi = A$.

(ii) $A \cap \phi = \phi$.

❖ ***Self Test IV*:**

For solving the exercises 1 to 5 below, consider the following sets:

$U = \{1,2,3,\ldots 10\}$, $A = \{x : x$ is a prime number less than $10\}$,

$B = \{2,4,6,8,10\}$ $C = \{1,4,9,16,25\}$,

Select the correct alternative from the given alternatives.

1. Which of the following is the set, $(A \cup B)^c$?

(a) $\{2, 3, 4, 5, 6, 7, 8, 10\}$ **(b)** $\{1, 9\}$ **(c)** $\{1,3,5,7,9\}$ **(d)** $\{1, 3, 5, 9\}$

2. Which of the following is the set, $A^c \cup B^c$?

(a) $\{1, 3, 4, 5, 6, 7, 8, 9, 10\}$ **(b)** $\{1, 2, 3, 4, 5\}$

(c) $\{1,3,5,7,9\}$ **(d)** $\{1, 9\}$.

3. Which of the following is the set, $((A)^c)^c$?

(a) $\{2, 3, 5, 7\}$ **(b)** $\{1,2,3,\ldots 10\}$ **(c)** $\{1, 4, 6, 8, 9, 10\}$ **(d)** $\{1, 2, 3, 5, 7\}$.

4. Which of the following is the set $C \cup A^c$?

(a) $\{2, 3, 4, 5, 6, 7, 8, 10, 16, 25\}$ **(b)** $\{1, 2, 3, 4, 5\}$

(c) $\{1,3,5,7,9\}$ **(d)** $\{1, 4, 6, 8, 9, 10, 16, 25\}$.

5. Which of the following is the set B^c?

(a) $\{3, 5, 7, 9\}$ **(b)** $\{1, 2, 3, \ldots, 10\}$

(c) $\{1, 3, 5, 7, 9\}$ **(d)** $\{2, 3, 5, 7, 9\}$.

6. Which of the following is an associative property ?

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(b)** $A \cup A = A$.

(c) $(A \cup B) \cup C = A \cup (B \cup C)$ **(d)** $(A \cap B)^c = A^c \cup B^c$.

7. Which of the following is a distributive property ?

(a) $(A \cap B)^c = A^c \cup B^c$ **(b)** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) $A \cup A = A$. **(d)** $(A \cup B) \cup C = A \cup (B \cup C)$.

8. Which of the following is an idempotent law?

(a) $A \cup A = A$. **(b)** $(A \cap B)^c = A^c \cup B^c$

(c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(d)** $(A \cup B) \cup C = A \cup (B \cup C)$.

9. Which of the following is a De Morgan's law??

(a) $(A \cup B) \cup C = A \cup (B \cup C)$ **(b)** $(A \cap B)^c = A^c \cup B^c$

(c) $A \cup A = A$. **(d)** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

10. Which of the following is not true?

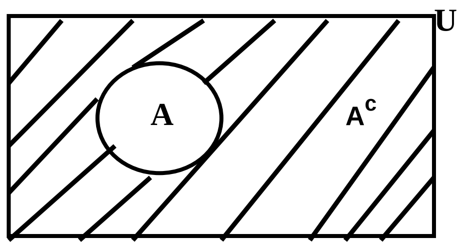
(a) $A \cup U = A$ **(b)** $\phi^c = U$ **(c)** $A \cap \phi = \phi$. **(d)** $A \cup A^c = U$.

2.6 Venn diagrams:

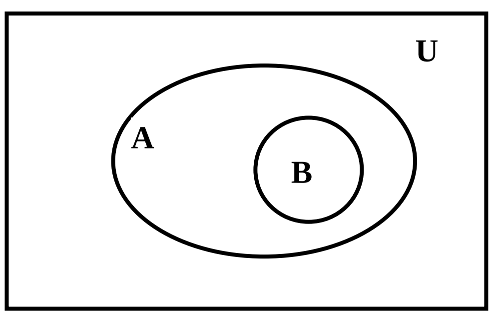
Sets can be represented, using Venn diagrams also. A Venn diagram is a pictorial representation of sets in a plane. In Venn diagrams sets are represented by circles, ellipses or closed curves and a rectangle. The Universal set is represented by the interior of a rectangle and the other sets are represented by circles or closed curves lying within the rectangle.

1. If U is universal set and $U \supset A$, then U is divided into two subsets A and A^c, Because $U = A \cup A^C$. A Venn diagram of this situation can be as shown below:

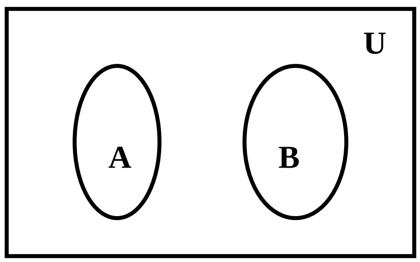
Shaded portion in this diagram shows A^C .



If $B \subset A$, then inside a rectangle representing universal set a circle or ellipse representing set A is drawn and the circle or ellipse representing the set B is drawn entirely within set A. It is because of the fact, that in this case every element of set B is also an element of set A. A Venn diagram of $B \subset A$ can be as shown below.



2. If A and B are disjoint sets i.e. A and B have no common elements then the circles or ellipses representing set A and set B are drawn separated within the rectangle representing universal set. A Venn diagram of this case can be as shown below:



3. If A and B are any two sets then it is possible that some elements of set U are in both sets A and B , some are only in set A but not in set B, some are only in set B but not in set A and some are neither in set A nor in set B. A Venn diagram of this case can be as shown below:
Coloured portion in the diagram 1 shows the set AUB and the coloured portion in this diagram 2 shows the set AUB. In these diagrams the two ovals represent sets A and B respectively and the rectangles represents the universal set.

Diagram1

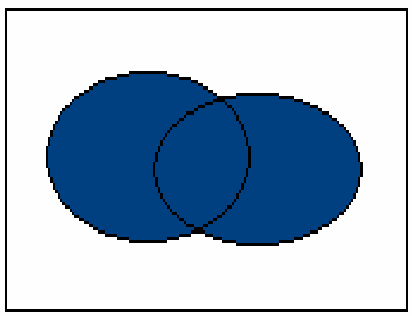
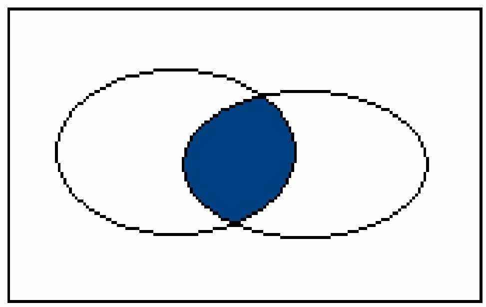


Diagram2



Self Test V

2.7 Summary for Unit 2

In this unit learners studied the following topics in details:

1. Concept of set and how to write sets using set listing form and set builder form.
2. Different types of sets which are null or empty set, singleton set, subset, superset, universal set , finite/ infinite set and power set etc.
3. How to perform various operations on sets such as union, intersection, difference and Cartesian product.
4. What are different properties of sets such as Commutative properties, Associative properties, Distributive properties, Idempotent laws, DeMorgan's laws, Properties of complements, Properties of the universal set and Properties of the null set.
5. Representation of sets using Venn diagrams.