UNIT 12 Mensuration

12.0 : Unit Objectives:

By the end of this Unit, learners should be able to:

- Compute the areas of plane figures such as triangle, rectangle and circle.
- Understand how to find Perimeter and circumference of above figures.
- Find surface areas of cube, cuboids, spheres and right circular cylinders.
- Compute Volumes of cube, cuboids, spheres and right circular cylinders.

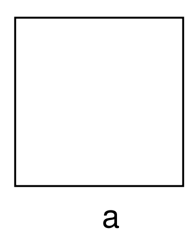
12.1 : Unit Introduction:

Mensuration in its literal meaning is the science of measurement. This word is generally used where geometrical figures are concerned and where one has to determine various physical quantities such as length, area and volume. Measurement is fundamental in science; it is one of the things that distinguish science from pseudoscience. Measurement is the process of estimating the magnitude of some attribute of an object, such as its length, weight, or depth relative to some standard unit of measurement. Measurement is also essential in industry, commerce, engineering, construction, manufacturing, pharmaceutical production, and electronics.

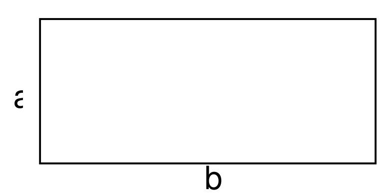
12.2 : Areas of plane figures:

Area is a quantity expressing the size of a figure typically a region bounded by a closed curve in the Euclidean plane or on a two dimensional surface. Points and lines have zero area. A figure may have infinite area, for example the entire Euclidean plane. Although area seems to be one of the basic notions in geometry, it is not easy to define even in the Euclidean plane.The expressions for the areas of some simple plane figures are as given below:

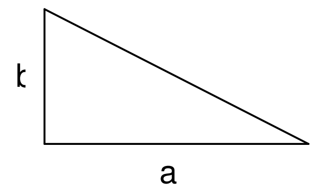
1. Square: The area A of a square of side length a units is given by, $A = a^2$ square units.



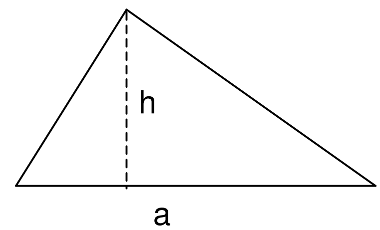
2. Rectangle: The area A of a rectangle of side lengths a and b i.e. height a and breadth b equals given by
$A = a \times b$ square units.



3. Right-angled triangle: The area A of a triangle having sides enclosing the right angle, of lengths a and b is given by, $A = \frac{1}{2} \times a \times b$ square units.



4. Triangle: The area A of a triangle with length of base a, and height h from the opposite angle is given by,

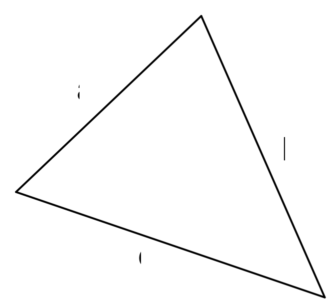


$A = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times a \times h$ square units.

Or

The area A of a triangle with lengths of the three sides a units, b units and c units respectively is given by ,

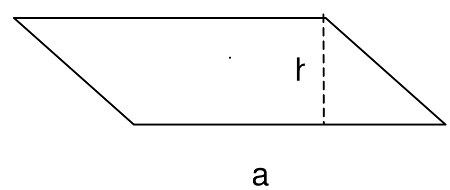


$A = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$ square units.

where $s = \frac{a+b+c}{2}$.

5. Parallelogram: The area A of a parallelogram with the two opposite sides a and perpendicular distance between them h is given by,

$A = h \times a$ square units.

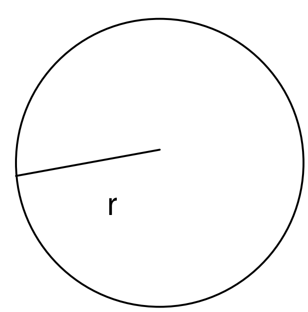


6. Circle: The mensuration of the circle is founded on the property that the areas of different circles are proportional to the squares on their diameters.

The area A of a circle is given by,

$A = \pi \times r^2$ square units, where r is the radius,

and π=3.14159 approximately.



Examples:

- Area of a square of side 5 cm. is equal to 25 cm^2.
- If ABCD is a rectangle with the length of side AB = 12cms. and the length of side AD = 21cm,

  Then the area A = length of side AB $\times$length of side AD

  $\therefore$ A = 12 $\times$21 = 252 cm^2.

- The area A of a triangle having height 32 mm. and length of base 65 mm

  $A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 32 \times 65 = 1040\ mm^2$.

- The area A of a triangle with sides 5 cm, 12 cm and 13 cm is,

  $A = \sqrt{s \times (s-5) \times (s-12) \times (s-13)}$.

  where $s = \frac{a+b+c}{2} = \frac{5+12+13}{2} = 15$

$\therefore A = \sqrt{15 \times (15-5) \times (15-12) \times (15-13)} = \sqrt{15 \times 10 \times 3 \times 2} = \sqrt{900} = 30\ cm^2$.

- The area A of a parallelogram with the length of two opposite sides 6cm and perpendicular distance between them 5cm is given by,

  $A = 6 \times 5 = 30$ square cm.

- The area A of a circle with radius 2 cm is,

  $A = \pi \times 2^2 = 4\pi = 12.5663\ cm^2$.

12.3 : Perimeters of plane figures:

The perimeter is the length of the line that bounds an area. The word may also be used to refer to the boundary line itself, but we will use perimeter as a measure. In the special case where the area is circular or elliptical the perimeter is called as the circumference.The perimeter of a polygon is the addition of the lengths of its sides. The expressions for the perimeters of some simple plane figures are given below:

1. Square: The perimeter P of a square of side length a is, P = 4$\times$a units.
2. Rectangle: The perimeter P of a rectangle of side lengths a and b i.e. say height a and breadth b equals is given by, P = 2 $\times$ (a + b) units.
3. Parallelogram: The perimeter P of a parallelogram with the two opposite sides a and b is given by, P = 2 $\times$ (a + b) units.

4. Circle: The perimeter P of a circle is called the circumference of the circle. It is given by $P = 2\times\pi\times r$ units, where r is the radius of the circle, and $\pi = 3.14159$ approximately.

Examples:

- The perimeter of a square of side 5 cm. is equal to $4\times 5 = 20$ cm.
- If ABCD is a rectangle with adjacent sides 12 cm and 21 cm in length length, then the perimeter $= 2\times(12+21) = 66$ cm.
- The perimeter of a parallelogram with the length of adjacent sides 32 mm. and 65 mm is,
  $P = 2\times(32+65) = 194$ mm.
- The perimeter of a triangle with sides 5 cm, 12 cm and 13 cm is,
  $P = a + b + c = 5 + 12 + 13 = 30$ cm.
- The perimeter i.e. circumference of a circle with radius 4 cm is,
  $P = 2\times\pi\times r = 8\pi = 25.1327$ cm.

Self Test I:

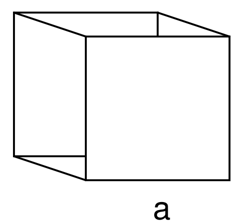
12.3 : Volumes of solid objects:

The 3-dimensional analog of area is the volume. The volume of any solid, plasma, vacuum or theoretical object is how much three dimensional space it occupies, often quantified numerically. One-dimensional figures and two-dimensional shapes) are assigned zero volume in the three-dimensional space. Volumes of straight-edged and circular shapes are calculated using arithmetic formulae.

Volume and capacity are sometimes distinguished, with capacity being used for how much a container can hold and volume being how much space an object displaces .The volume of a dispersed gas is the capacity of its container. Volume and capacity are also distinguished in a capacity management setting, where capacity is defined as volume over a specified time period.

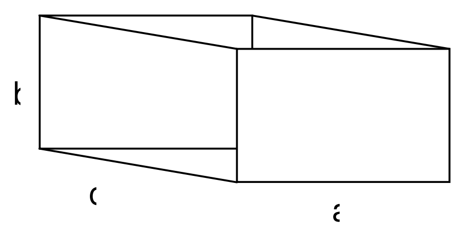
The expressions for the volumes of some simple figures in three dimensional space are as given below:

1. Cube: A cube is a three dimensional solid object bounded by six square faces or sides, with three meeting at each vertex. All the faces are perpendicular to each other. Volume V of a cube of side length a units of a square face is given by, V= a^3 cubic units.

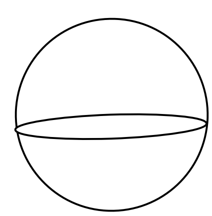


2. Cuboids: A cuboid or a rectangular paralleopiped is a three dimensional solid object bounded by six rectangular faces or sides, with three meeting at each vertex. All the faces are perpendicular to each other. Volume V of a cuboid of side lengths a , b and c units respectively is given by , V = length $\times$ height $\times$ width

= a $\times$ b $\times$ c cubic units.



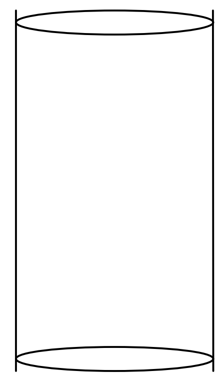
3. Sphere: A sphere is a symmetrical geometrical object. In non-mathematical usage, the term is used to refer either to a round ball or globe. In mathematics, a sphere is the set of all points in three-dimensional space which are at distance *r* from a fixed point of that space,



where *r* is a positive real number called the radius of the sphere and the fixed is centre of the sphere. The volume Vof a sphere of radius r is given by ,

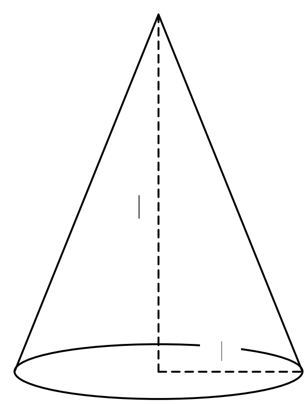
$V=\frac{4}{3} \times \pi \times r^3$ cubic units.

4. Right circular cylinder: A cylinder is bounded by two parallel planes and by the surface generated by a line segment rotating parallel to itself. The parallel planes are called the bases. A right circular cylinder is generated by revolving a rectangle about one of its sides as an axis. For a right circular cylinder the bases are circles. The height of a cylinder is the perpendicular distance between the bases. If the right circular cylinder has a radius r and height h, then its volume V is given by, $V= \pi \times r^2 \times h$ cubic units.



5. Right circular cone:

A right circular cone is generated by revolving a right angled triangle about one of its sides as an axis. For a right circular cone its base is a circle. The height of a right circular cone is the perpendicular from the vertex to the centre of the circular base. If the right circular cone has height h and the radius of the circular base r , then its volume V is given by , $V= \frac{1}{3} \times \pi \times r^2 \times h$ cubic units.



Examples:

- Volume of a cube of side 5 cm. is equal to 125 cm^3.
- Volume of a rectangular parallelepiped with length 30cm, height 5 cm and width 20 cm, is

  $V = \text{length} \times \text{height} \times \text{width}.$

  $\therefore A = 30 \times 5 \times 20 = 3000 \; cm^3.$
- Volume of a sphere with radius 5 cm is

$$V = \frac{4}{3} \times \pi \times r^3 = \frac{4}{3} \times \pi \times 5^3$$

$$\therefore A = \frac{4}{3} \times \pi \times 125 = 523.5987 \quad cm^3.$$

- Volume of a right circular cylinder with radius 4.6 cm and height 8.5cm, is given by, V= $\pi \times (4.6)^2 \times 8.5 = 565.0468 \quad cm^3$.
- Volume of right circular cone that has height 20 cm and the radius of the circular base 15 cm , then its volume V = $\frac{1}{3} \times \pi \times (15)^2 \times 20$ $= 4712.3889 \quad cm^3$.

12.4: Surface areas solid objects:

In general, the surface area is the sum of all the areas of all the shapes that cover the surface of the object. The term surface area refers to the total area of the exposed surface of a 3-dimensional solid, such as the sum of the areas of the exposed sides of a polyhedron. Surface area is the measure of how much exposed area an object has. It is expressed in square units. If an object has flat faces, its surface area can be calculated by adding together the areas of its faces. Even objects with smooth surfaces, such as spheres, have surface area.

The expressions for the surface areas of some simple figures are as given below:

1. Cube: As a cube is a solid object bounded by six square faces, with three of them meeting at each vertex. All square faces have equal area which is given by a^2 square units.

$\therefore$ The surface area S of a cube of side length a units is, S = $6 \times a^2$ square units.

2. Cuboids: A cuboid is a three dimensional solid object bounded by six rectangular faces with three meeting at each vertex. The faces apposite to each other have equal area, which can be obtained by length $\times$ height formula for area of a rectangle

∴ The surface area S of a cuboid of side lengths a , b and c units respectively is given by,

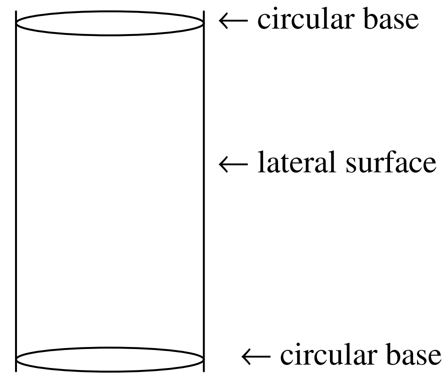
$$S = 2\times \text{length} \times \text{height} + 2\times \text{height} \times \text{width} + 2\times \text{length} \times \text{width}$$
$$= (2\times a\times b + 2\times b \times c + 2\times a \times c) \text{ square units.}$$

3. Sphere: The surface area S of a sphere of radius r is four times the surface area of a great circle of the sphere.

∴ The surface area S of a sphere of radius r is given by,

$$S = 4 \times \pi \times r^2 \text{ square units.}$$

4. Right circular cylinder:



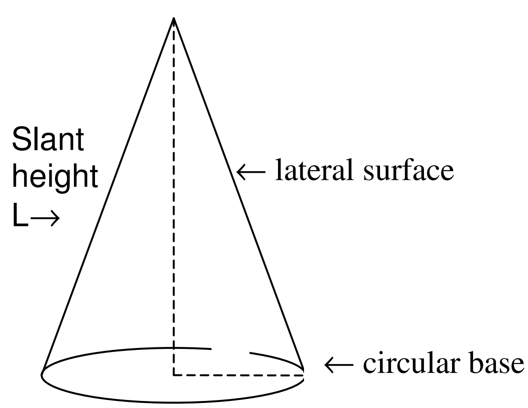
The surface area S of right circular cylinder is sum of the lateral surface area and the area of two bases. For a right circular cylinder if the circular base has radius r. Then the area of the two bases is given by $2 \times \pi \times r^2$.

If the right circular cylinder has and height h, then the lateral surface area is given by $2 \times \pi \times r \times h$.

∴ The surface area $S = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

$$= 2 \times \pi \times r \times (r + h) \text{ square units.}$$

5. Right circular cone:



The surface area S of right circular cone is sum of the lateral surface area and the area of the circular base. For a right circular cone if the circular base has radius r. Then the area of base is given by $\pi \times r^2$.

If the right circular cone has slant height L, then the lateral surface area is given by $\frac{1}{2} \times 2 \times \pi \times r \times L = \pi \times r \times L$.

∴ The surface area $S = \pi \times r^2 + \times \pi \times r \times L$

$$= \pi \times r \times (r + L) \text{ square units.}$$

Note that , if the right circular cone has slant height L, the circular base has radius r and the vertical height is h then by Pythagoras theorem, we have

$L^2 = r^2 + h^2$ or $L = \sqrt{r^2 + h^2}$

Examples:

- Surface area of a cube of side 5 cm. is equal to $6 \times 25 = 150$ cm^2.
- Surface area S, of a rectangular parallelepiped with length 30cm, height 5 cm and width 20 cm, is

  $S = 2 \times (\text{length} \times \text{height} + \text{height} \times \text{width} + \text{length} \times \text{width})$.

  $\therefore S = 2 \times (30 \times 5 + 5 \times 20 + 30 \times 20) = 1700$ cm^2.
- Surface area S of a sphere with radius 5 cm is

  $S = 4 \times \pi \times r^2 = 4 \times \pi \times 5^2$

  $\therefore A = 314.1592$ cm^2.
- Surface area S of a right circular cylinder with radius 4.6 cm and height 8.5cm, is given by,

  $S = 2 \times \pi \times r \times (r + h) = 2 \times \pi \times 4.6 \times (4.6 + 8.5)$

  $= 378.6247$ cm^2.
- Surface area S of right circular cone that has height 20 cm and the radius of the circular base 15 cm , then its $S = \pi \times r^2 + \times \pi \times r \times L$

  where $L = \sqrt{r^2 + h^2} = \sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25$ cm

$\therefore$ $S = \pi \times 15 \times (15 + 25) = 1884.9555$ cm^2.

Self Test II:

12.5 :Summary for Unit 12:

In this unit learners studied the following topics in details:

1. The areas of plane figures such as triangle, rectangle and circle.

2. Perimeter and circumference of plane figures.

3. Volumes of cube, cuboids, spheres and right circular cylinders.

4. Surface areas of cube, cuboids, spheres and right circular cylinders