UNIT 13 System of Linear Equations

13.0 : Unit Objectives:

By the end of this Unit, learners should be able to:

- Define Linear equation.
- Understand and explain system of linear equations
- Find Solution of System of linear equations
- Apply Cramer's Rule to find solution of a system containing upto 3 variables and 3 equations.

13.1 : Unit Introduction:

The equations which can be used to represent a straight line in a plane or in three dimensional space are linear equations. Besides geometry, there are other fields which involve a linear relationship between different unknown quantities or variables. The study of linear equations is important and very useful for solving problems related with speed and time , age, profit and loss etc.

We will study system of linear equations containing upto 3 variables.

13.2 : Linear equations:

Consider the following problem: If the cost of 2 pens and 3 pencils is 26 Rs. and the cost of 3 pens and 2 pencils is 34 Rs., the what is the cost of one pen and one pencil respectively?

To solve such problems we use linear equations. For the values we want to find we assume some unknowns or variables. Using these variables we convert the given information in equations and then solve them. Before solving this problem let us define linear equation formally.

Definition 13.2.1: Linear equation :A linear equation in variables $x_1, x_2, x_3, \ldots, x_n$ is defined as

$a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_nx_n = b$, where $a_1, a_2, a_3, \ldots, a_n$ and b are real numbers . And a1, $a_2, a_3, \ldots, a_n$ are called coefficients and b is called the constant term in this linear equation.

Note that all variables in a linear equation occur with power 1 only and no multiplication or division of variables occurs in it.

Examples:

- $3x_1 + 7x_2 – x_3 = – 3$, is a linear equation in 3 variables.

- Equation of any line in plane is a linear equation.
- $8x - y + 4z + \sqrt{6}\,w = 4$, is a linear equation in 4 variables.
- $3\,x_1 \times x_2 - 3x_3 = 14$, is not a linear equation as the first term involves multiplication of variables.
- $7x = 12y$ is a linear equation in 2 variables.
- $4x + 77y - 4\sqrt{z} = -\dfrac{5}{21}$, is not a linear equation.

Definition 13.2.2: <u>Solution of a linear equation :</u>A solution of linear equation $a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_nx_n = b$, is a sequence of n numbers $s_1, s_2, s_3, \ldots, s_n$ such that the equation is satisfied when we substitute $x_1 = s_1$, $x_2 = s_2$, $x_3 = s_2$, $\ldots$, $x_n = s_n$, That is $a_1s_1 + a_2s_2 + a_3s_3 + \ldots + a_ns_n = b$.

The set of all solutions is called the solution set of the linear equation.

Examples:

- $3x_1 + 7x_2 - x_3 = -3$, is a linear equation, it is satisfied by the values $x_1 = 1$, $x_2 = -1$, $x_3 = -1$ because $3 \times 1 + 7 \times (-1) - (-1) = 3 - 7 + 1 = -3$.

  Hence 1, –1, –1 is a solution to the linear equation $3x_1 + 7x_2 - x_3 = -3$.

  Note that this is not the only possible solution of this equation.
- $8x - y + 4z + \sqrt{6}\,w = 4$, is a linear equation which is satisfied when we substitute $x = 2$, $y = \sqrt{6}$, $z = 3$ and $w = 1$, because

$$(8 \times 2) - (\sqrt{6}) + (4 \times 3) + (\sqrt{6} \times 1) = 4$$

So the equation $8x - y + 4z + \sqrt{6}\,w = 4$ has a solution 2, $\sqrt{6}$, 3 , 1.

Note that this is not the only possible solution of this equation. There exist other solutions also.

<u>***Self Test I:***</u>

13.3 : System of linear equations

Definition 13.3.1: System of linear equation :A finite set of linear equations in variables $x_1, x_2, x_3, \ldots, x_n$ is called a system of linear equations, in these variables.

In general a system of m equations in n variables $x_1, x_2, x_3, \ldots, x_n$ is written as:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n = b_3$

. . . .

:

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n = b_m$

Definition 13.3.2: Homogeneous/nonhomogeneous system of linear equation :

A system of linear equations in variables $x_1, x_2, x_3, \ldots, x_n$ is called a homogeneous system of linear equations if the constant term of each equation of the system is zero.

A system is called as a non homogeneous system if it is not a homogeneous system.

In general a homogeneous system of m equations in n variables $x_1, x_2, x_3, \ldots, x_n$ is written as:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = 0$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = 0$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n = 0$

. . . .

:

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n = 0.$

Examples:

- $2x_1 - 3x_2 = -1$

  $x_1 + x_2 = 2$

  $x_1 - x_2 = 0.$

is a system of linear equations in 2 variables x_1 and x_2.This system is containing 3 equations. It is non homogeneous system.

- $x + 3y + 5z = 0$

  $3x + 2y + 7z = 0$

$4x - 7y - 3z = 0$

It is a homogeneous system of linear equations in 3 variables x, y $_{\text{and}}$ z. This system is containing 3 equations.

- $x + y + 2z = 7$
  $-x - 2y + 3z = 6$
  $3x - 7y + 6z = 1$

It is a non homogeneous system of linear equation in 3 variables x, y and z, it contains 3 equations.

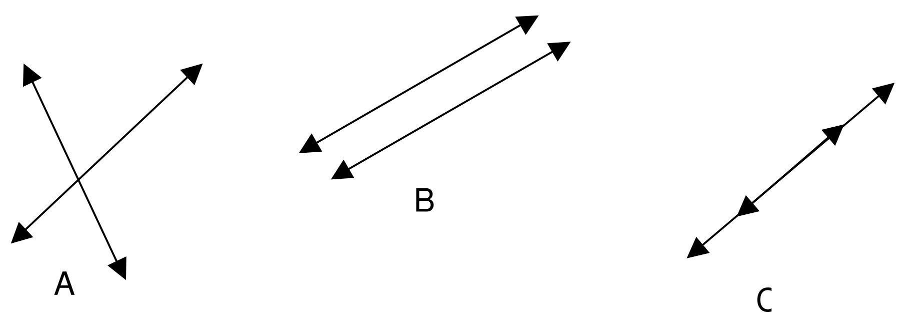
Definition 13.3.3: Solution of system of linear equation :A sequence of n numbers $s_1, s_2, s_3, \ldots, s_n$ is a solution of the system of linear equation if $x_1 = s_1$, $x_2 = s_2$, $x_3 = s_2$, …, $x_n = s_n$, is a solution of each equation in the system.

The set of all solutions is called the solution set of the system of linear equation.

Note that :

A system of linear equations has either a unique solution, or infinitely many solutions or no solution.

It is easy to explain this geometrically, if we consider a set of 2 straight lines on plane (or in space),then one of the following situations will occur.



A) In this situation the two lines are intersecting at a point. And the corresponding linear equations form a system, which has an unique solution.

B) In this situation the two lines are parallel they are not intersecting each other. And the corresponding linear equations form a system, which has no solution.

C) In this situation the two lines are coincident, i.e. they are intersecting at every point. And the corresponding linear equations form a system, which has infinitely many solutions.

Examples:

- Consider the system of equations
  $x + 2y = 3$
  $3x + 2y = 5$

$x - y = 0$

Each equation of this system is satisfied by the values $x = 1$, $y = 1$

$\therefore$ 1, 1 is a solution of this system. It is a unique solution to this system.

- $x + y = 2$
  $5x + 5y = 10$.

Each equation of this system is satisfied by the values $x = t$, $y = 2 - t$ where t is a real number.
e.g. $x = 1$, $y = 1$ is a solution of this system,
also $x = 2$, $y = 0$ is a solution of this system, $x = 3$, $y = -1$ is a solution of this system etc . $\therefore$ This system has infinitely many solutions.

- $x + y = 2$
  $5x + 5y = 11$.

In this case from first equation we get $y = 2 - x$, substituting it in the other equation we get

$$5x + 5(2 - x) = 11$$

$$\therefore 5x + 5 \times 2 - 5x = 11$$

$\therefore$ $10 = 11$ which is not possible hence it is not possible that $y = 2 - x$,

$\therefore$ This system has no solution.

Definition 13.3.4: Consistent / inconsistent system: A system of linear equations is called as a consistent system if it has at least one solution otherwise i.e. if it has no solution then it is called as an inconsistent system .

Examples:

- $x + 3y + 5z = 0$
  $3x + 2y + 7z = 0$
  $4x - 7y - 3z = 0$

Each equation of this system is satisfied by the values $x = 0$, $y = 0$ and $z = 0$.

$\therefore$ 0, 0, 0 is a solution of this system.

$\therefore$ This system is a consistent system.

Note that in general a homogeneous system is always consistent system, because 0,0,0,….0 is always solution of it.

- $2x_1 - 3x_2 = -1$
  $x_1 + x_2 = 2$
  $x_1 - x_2 = 0.$

is a system of linear equation. Each equation of this system is satisfied by the values $x_1 = 1$, $x_2 = 1$.

$\therefore$ 1, 1 is a solution of this system.

$\therefore$ This system is a consistent system.

- $$\begin{aligned} x + y + 2z &= 7 \\ -x - 2y + 3z &= 6 \\ 3x - 7y + 6z &= 1 \end{aligned}$$

Each equation of this system is satisfied by the values $x = -1$, $y = 2$ and $z = 3$.

$\therefore$ $-1, 2, 3$ is a solution of this system.

$\therefore$ This system is a consistent system.

13.3.1: Representation of system of equations in matrix form:

Consider the general system of m equations in n variables $x_1, x_2, x_3, \ldots, x_n$ written as:

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n &= b_3 \\
\ldots & \\
: & \\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n &= b_m
\end{aligned}$$

Using matrix we can write this system as ,

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots + a_{3n}x_n = b_3 \\ \ldots \\ : \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n = b_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ : \\ : \\ b_m \end{bmatrix}$$

This is equivalent to the matrix multiplication,

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

or A X = B where A= $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$, X = $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and B = $\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$

So the general system of m equations in n variables has the matrix form

A X = B where A= $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ is called the matrix of coefficients,

X = $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ is called the matrix of variables and

B = $\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$ is called the matrix of constant terms.

Note that :

1. If A is invertible then the system AX = B has unique solution.
2. There are different methods to solve the system AX = B.
3. If A is invertible matrix then solution of AX = B is X = A^{-1} B .
4. One method to find solution of system of linear equations is Cramer's rule which we will study later in this unit.

Examples:

- The system of linear equations

$$x + 3y + 5z = 0$$
$$3x + 2y + 7z = 0$$
$$4x - 7y - 3z = 0$$

has the matrix form A X = B where ,

the matrix of coefficients, A = $\begin{bmatrix} 1 & 3 & 5 \\ 3 & 2 & 7 \\ 4 & -7 & -3 \end{bmatrix}$,

the matrix of variables X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and the matrix of constant terms, B = $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

- The system of linear equations

$$2x_1 - 3x_2 = -1$$
$$x_1 + x_2 = 2$$
$$x_1 - x_2 = 0.,$$ has the matrix form A X = B where ,

the matrix of coefficients is , A = $\begin{bmatrix} 2 & -3 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$, the matrix of variables is X = $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

and the matrix of constant terms is, B = $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$.

Self Test II:

13.3 : Cramer's Rule

If a system of n linear equations and n unknowns is such that the matrix of coefficients is invertible then this system has unique solution. One method of solving such system is called the method of Cramer's Rule, it is based on the use of determinants. To solve any system of linear equation using Cramer's rule we need to write the system in its matrix form.

13.3.1: Cramer's Rule: (Statement): If A X = B is a matrix form of the system of n linear equations in n variables, such that the determinant $|A| \neq 0$, then the system has a unique solution given by the formula,

$x_i = \dfrac{|A_i|}{|A|}, 1 \le i \le n$, where A_i is the matrix obtained from A by replacing the elements in i^{th} column by the entries in the matrix of constants B.

Note that the system in n variables and n equations only can be solved using this rule. A system for which coefficient matrix is not a square matrix can not be solved using Cramer's rule.

1. Cramer's rule for a system of 2 equation in 2 variables is as below:

If a system is $a_1 x + b_1 y = c_1$

$a_2 x + b_2 y = c_2$ then in its matrix form,

where, the matrix of coefficients is , A = $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$,

the matrix of variables is X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and

the matrix of constant terms is, B = $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.

In this case the solution by the Cramer's rule is , $x = \dfrac{|A_1|}{|A|}$, $y = \dfrac{|A_2|}{|A|}$.

where $|A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $|A_1| = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ and $|A_2| = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$.

Example:

- Consider the problem with which we started our discussion: If the cost of 2 pens and 3 pencils is 26 Rs. and the cost of 3 pens and 2 pencils is 34 Rs., the what is the cost of one pen and one pencil respectively?

Solution : To solve this problem assume that the cost of a single pen is x Rs. and the cost of a single pencil is y Rs.

The information that "the cost of 2 pens and 3 pencils is 26 Rs." is now converted to the equation 2x + 3y = 26.

And the information that "the cost of 3 pens and 2 pencils is 34 Rs." is now converted to the equation 3x + 2y = 34 .

We want to find x and y satisfying above two equations. That means we want to find solution of the following system in two variables x and y.

2x + 3y = 26

3x + 2y = 34

In matrix form the system is A X = B where the matrix of coefficients is , A = $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$,

the matrix of variables is X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and

the matrix of constant terms is, B = $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 26 \\ 34 \end{bmatrix}$.

In this case the solution by the Cramer's rule is , x = $\frac{|A_1|}{|A|}$, y = $\frac{|A_2|}{|A|}$ ....(1)

where $|A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 2 \times 2 - 3 \times 3 = 4 - 9 = -5$.

$|A_1| = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = \begin{vmatrix} 26 & 3 \\ 34 & 2 \end{vmatrix} = 26 \times 2 - 34 \times 3 = 52 - 102 = -50$.

and $|A_2| = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 26 \\ 3 & 34 \end{vmatrix} = 2 \times 34 - 3 \times 26 = 68 - 78 = -10$.

$\therefore$ by the Cramer's rule x = $\frac{|A_1|}{|A|} = \frac{-50}{-5} = 10$

and y = $\frac{|A_2|}{|A|} = \frac{-10}{-5} = 2$

$\therefore$ Solution is x = the cost of a pen is 10 Rs. and y = the cost of a pencil is 2 Rs.

2. Cramer's rule for a system of 3 equation in 3 variables is as below:

If a system is
$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

then its matrix form is A X = B where,

the matrix of coefficients is , A = $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a3 & b_3 & c_3 \end{bmatrix}$,

the matrix of variables is X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and

the matrix of constant terms is, B = $\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$.

In this case the solution by the Cramer's rule is,

x = $\dfrac{|A_1|}{|A|}$, y = $\dfrac{|A_2|}{|A|}$, z = $\dfrac{|A_3|}{|A|}$.

where| A| = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a3 & b_3 & c_3 \end{vmatrix}$, $|A_1|$ = $\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d3 & b_3 & c_3 \end{vmatrix}$, $|A_2|$ = $\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a3 & d_3 & c_3 \end{vmatrix}$

and $|A_3|$ = $\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a3 & b_3 & d_3 \end{vmatrix}$.

Examples:

- Solve the following system of 3 equation in 3 variables, by Cramer's rule

$$x + y + 2z = 7$$
$$-x - 2y + 3z = 6$$
$$3x - 7y + 6z = 1$$

<u>Solution:</u> The given system has matrix form, A X = B where,

the matrix of coefficients is , A = $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ = $\begin{bmatrix} 1 & 1 & 2 \\ -1 & -2 & 3 \\ 3 & -7 & 6 \end{bmatrix}$

the matrix of variables is X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and the matrix of constant terms is, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ 1 \end{bmatrix}$

In this case the solution by the Cramer's rule is,

$x = \dfrac{|A_1|}{|A|}$, $y = \dfrac{|A_2|}{|A|}$, $z = \dfrac{|A_3|}{|A|}$ ....(1)

where $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 \\ -1 & -2 & 3 \\ 3 & -7 & 6 \end{vmatrix} =$

$= 1\times [(-2 \times 6) - (-7 \times 3)] - 1\times[(-1 \times 6) -(3 \times 3)] + 2\times[(-1 \times -7) -(3 \times -2)]$

$= 1\times [(-12) - (-21)] - 1\times[(-6) -(9)] + 2\times[7 + 6]$

$= 1\times [9] - 1\times[-15] + 2\times[13]$

$= 9 + 15 + 26 = 50$

$|A_1| = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 7 & 1 & 2 \\ 6 & -2 & 3 \\ 1 & -7 & 6 \end{vmatrix}$

$= 7\times [(-2 \times 6) - (-7 \times 3)] - 1\times[(6 \times 6) -(1 \times 3)] + 2\times[(6 \times -7) -(1 \times -2)]$

$= 7\times [(-12) - (-21)] - 1\times[(36) -(3)] + 2\times[-42 + 2]$

$= 7\times [9] - 1\times[33] + 2\times[-40]$

$= 63 - 33 - 80 = -50$

$|A_2| = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \begin{vmatrix} 1 & 7 & 2 \\ -1 & 6 & 3 \\ 3 & 1 & 6 \end{vmatrix}$

$= 1\times [(6 \times 6) - (1 \times 3)] - 7\times[(-1 \times 6) -(3 \times 3)] + 2\times[(-1 \times 1) -(3 \times 6)]$

$= 1\times [(36) - (3)] - 7\times[(-6) -(9)] + 2\times[-1 - 18]$

$= 1\times [33] - 7\times[-15] + 2\times[-19]$

$= 33 + 105 - 38 = 100$

and $|A_3| = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 7 \\ -1 & -2 & 6 \\ 3 & -7 & 1 \end{vmatrix} =$

$= 1\times [(-2 \times 1) - (-7 \times 6)] -1\times[(-1 \times 1) -(3 \times 6)] +7\times[(-1 \times -7) -(3 \times -2)]$

$=1\times [(-2) - (-42)] -1\times[(-1) -(18)] +7\times[7 +6]$

$= 1\times [40] -1\times[-1\ 9] +7\times[13]$

$= 40 + 19 + 91 = 150$

$\therefore$ By (1) $x = \dfrac{|A_1|}{|A|} = \dfrac{-50}{50} = -1$

$y = \dfrac{|A_2|}{|A|} = \dfrac{100}{50} = 2$

and $z = \dfrac{|A_3|}{|A|} = \dfrac{150}{50} = 3$

$\therefore$ Solution of the system is $x = -1, \quad y = 2 \quad, z = 3$

$\therefore$ Solution set of the system = $\{ -1, \ 2 \ , 3 \}$.

<u>Self Test III:</u>

13.4:Summary for Unit 13:

In this unit learners studied the following topics in details:

1. The concept of Linear equation, solution of linear equation.

2. System of linear equations and solution of system of linear equations.

3. Matrix representation of system of linear equations.

4. Cramer's Rule to find solution of a system containing upto 3 variables and 3 equations.